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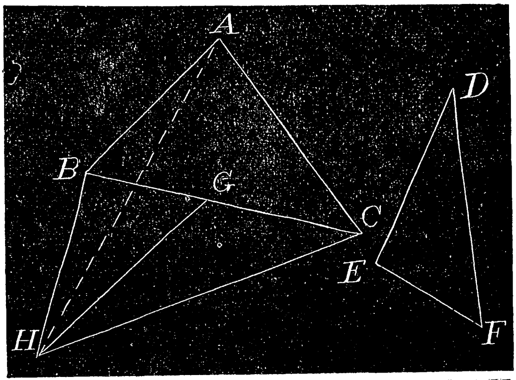
DEMONSTRATION OF PROP. XXV. OF THE FIRST BOOK OF EUCLID.

BY W. E. HEAL, WHEELING, INDIANA.

THE twenty-fifth proposition of the first book of Euclid, which that author (and, I believe, every succeeding geometer) demonstrates indirectly, that is by the *reductio ad absurdum*, may be demonstrated directly as follows:

Theorem:—If two triangles have two sides of the one equal to two sides of the other, each to each, but the bases unequal, the angle contained by the sides of that which has the greater base will be greater than the angle contained by the sides of the other.

In the triangles ABC, DEF let $AB = DE$, $AC = DF$ and BC be greater than EF, then shall the angle BAC be greater than the angle EDF.

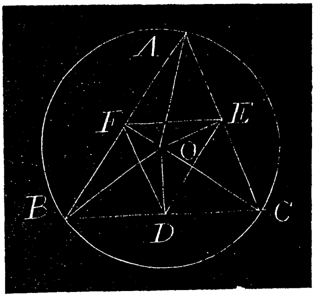


Of the sides AB, AC let AB be the one which is not greater than the other, and from the base BC cut off a part $BG = EF$ and adjacent to AB; on BG construct the triangle BHG so that $BH = DE$ (or AB) and $HG = DF$ (or AC); join AH and HC. Because $BH = BA$, the angle BHA = the angle BAH. And since the side BH is not greater than HG the angle HGB is not greater than HBG; but the angle HGC is greater than HBG; therefore the angle HGC is greater than HGB; but HGB is greater than HCG; much more, then, is the angle HGC greater than HCG. And because the angle HGC is gaeater than HCG the side HC is greater than the side HG or its equal AC. Then, in the triangle HCA, the angle HAC is greater than the angle AHC because the side HC is greater than AC. To these unequal angles add the equals BHA and BAH and there results angle BAC greater than angle BHC; but $BHG = EDF$; therefore the angle BAC is greater than the angle EDF; which was to be proved.

QUERY, BY THE EDITOR. — As we have been requested to furnish a demonstration to the following propositon, the subjoined demonstration is submitted; and we present to our readers the query: By whom was the proposition originally announced?—Also, give the author's demonstration.

Proposition.—"$D+d = a\cot A + b\cot B + c\cot C$, in which d and D are the diameters of the circles inscribed in, and circumscribed about, a triangle whose sides are a, b, c, and opposite angles A, B, C."

Demonstration.—Let ABC represent the triangle and O, the center of the circumscribing circle, and let D, E, F, represent the middle points of the sides a, b, c.



Because the $\angle BOC$ at the center is double the $\angle A$ at the circumference, $\angle BOD = \angle A$.

$$\therefore \quad DO\ (=x) = \tfrac{1}{2}a\cot A, \text{ and similarly}$$
$$EO\ (=y) = \tfrac{1}{2}b\cot B,$$
$$FO\ (=z) = \tfrac{1}{2}c\cot C.$$

Join DE, DF and EF; then is $EF = \frac{1}{2}a$, $DF = \frac{1}{2}b$ and $DE = \frac{1}{2}c$.

Because the quadrilaterals $DCEO$, $EAFO$ and $FBDO$ have, each, the sum of their opposite angles $= 180°$ they are inscribable in a circle:

Therefore (Eucl. D. VI.) we have

$$\tfrac{1}{2}aR = \tfrac{1}{2}cy + \tfrac{1}{2}bz,$$
$$\tfrac{1}{2}bR = \tfrac{1}{2}cx + \tfrac{1}{2}az,$$
$$\tfrac{1}{2}cR = \tfrac{1}{2}ay + \tfrac{1}{2}bx.$$
$$\therefore \quad R(a+b+c) = (b+c)x + (a+c)y + (a+b)z, \tag{1}$$

where R is the radius of the circumscribing circle.

Let r represent the radius of the inscribed circle, then, obviously,

$$r(a+b+c) = ax + by + cz. \tag{2}$$

Adding (1) and (2) we get

$$(a+b+c)(R+r) = (a+b+c)x + (a+b+c)y + (a+b+c)z,$$

or

$$R + r = x + y + z = \tfrac{1}{2}a\cot A + \tfrac{1}{2}b\cot B + \tfrac{1}{2}c\cot C.$$
$$\therefore \quad D+d = a\cot A + b\cot B + c\cot C.$$

SOLUTIONS OF PROBLEMS IN NUMBER THREE.

SOLUTIONS of problems in No. 3 have been received as follows:

From Marcus Baker, 162 and 164; Prof. L. G. Barbour, 162; Prof. W. P. Casey, 166; George Eastwood, 164 and 166; Prof. H. T. Eddy, 165 & 168; Henry Gunder, 162; W. E. Heal, 162; Chas. H. Kummell, 162, 168 and 169; Prof. O. Pratt, Sen., 163; J. W. P. Reid, 162; L. Regan, 162; E. B. Seitz, 162 and 164; Prof. C. M. Woodward, 167; A. W. Whitaker, 162; F. A. Walker, 162.

162. "Given

$$x^2 + xy + y^2 = 37, \ (1)$$
$$x^2 + xz + z^2 = 49, \ (2)$$
$$y^2 + yz + z^2 = 61, \ (3)$$

to find x, y and z by quadratics".